arXiv:hep-ph/9911202v1 2 Nov 1999

October, 1999

Factorization Approach for the $\Delta I = 1/2$ Rule and ε'/ε in Kaon Decays

Hai-Yang Cheng

Institute of Physics, Academia Sinica,
Taipei, Taiwan 115, Republic of China

Abstract

The $\Delta I = 1/2$ rule and direct CP violation ε'/ε in kaon decays are studied within the framework of the effective Hamiltonian approach in conjunction with generalized factorization for hadronic matrix elements. We identify two principal sources responsible for the enhancement of $\mathrm{Re}A_0/\mathrm{Re}A_2$: the vertex-type as well as penguin-type corrections to the matrix elements of four-quark operators, which render the physical amplitude renormalization-scale and -scheme independent, and the non-factorized effect due to soft-gluon exchange, which is needed to suppress the $\Delta I = 3/2$ $K \to \pi\pi$ amplitude. Contrary to the chiral approach which is limited to light meson decays and fails to reproduce the A_2 amplitude, the aforementioned approach for dealing with scheme and scale issues is applicable to heavy meson decays. We obtain $\mathrm{Re}A_0/\mathrm{Re}A_2 = 13 - 15$ if $m_s(1\,\mathrm{GeV})$ lies in the range (125−175) MeV. The bag parameters B_i, which are often employed to parametrize the scale and scheme dependence of hadronic matrix elements, are calculated in two different renormalization scehemes. It is found that $B_8^{(2)}$ and $B_6^{(0)}$, both of order 1.5 at $\mu = 1$ GeV, are nearly γ_5 scheme independent, whereas $B_{3,5,7}^{(0)}$ as well as $B_7^{(2)}$ show a sizable scheme dependence. Moreover, only $B_{1,3,4}^{(0)}$ exhibit a significant m_s dependence, while the other B-parameters are almost m_s independent. For direct CP violation, we obtain $\varepsilon'/\varepsilon = (0.7 - 1.1) \times 10^{-3}$ if $m_s(1\,\mathrm{GeV}) = 150$ MeV and $\varepsilon'/\varepsilon = (1.0 - 1.6) \times 10^{-3}$ if m_s is as small as indicated by some recent lattice calculations.

I. INTRODUCTION

The celebrated $\Delta I = 1/2$ rule in kaon decays still remains an enigma after the first observation more than four decades ago. The tantalizing puzzle is the problem of how to enhance the A_0/A_2 ratio of the $\Delta I = 1/2$ to $\Delta I = 3/2$ $K \to \pi\pi$ amplitudes from the outrageously small value 0.9 [see Eq. (5.1) below] to the observed value 22.2 ± 0.1 (for a review of the $\Delta I = 1/2$ rule, see [1]). Within the framework of the effective weak Hamiltonian in conjunction with the factorization approach for hadronic matrix elements, the A_0/A_2 ratio is at most of order 8 even after the nonfactorized soft-gluon effects are included [1]. Moreover, the μ dependence of hadronic matrix elements is not addressed in the conventional calculation. In the past ten years or so, most efforts are devoted to computing the matrix elements to $\mathcal{O}(p^4)$ in chiral expansion. This scenario has the advantages that chiral loops introduces a scale dependence for hadronic matrix elements and that meson loop contributions to the A_0 amplitude are large enough to accommodate the data. However, this approach also exists a fundamental problem, namely the long-distance evolution of meson loop contributions can only be extended to the scale of order 600 MeV, whereas the perturbative evaluation of Wilson coefficients cannot be reliably evolved down to the scale below 1 GeV. The conventional practice of matching chiral loop corrections to hadronic matrix elements with Wilson coefficient functions at the scale $\mu = (0.6 - 1.0)$ GeV requires chiral perturbation theory and/or perturbative QCD be pushed into the regions beyond their applicability.

Another serious difficulty with the chiral approach is that although the inclusion of chiral loops will make a large enhancement for A_0, it cannot explain the A_2 amplitude. For example, in the analysis of [2] in which a physical cutoff Λ_c is introduced to regularize the quadratic and logarithmic divergence of the long-distance chiral loop corrections to $K \to \pi\pi$ amplitudes, the amplitude A_2 is predicted to be highly unstable relative to the cutoff scale Λ_c and it even changes sign at $\Lambda_c \gtrsim 650$ MeV [2,3]. In the approach in which the dimensional regularization is applied to regularize the chiral loop divergences and to consistently match the logarithmic scale dependence of Wilson coefficients, the predicted A_2 amplitude is too large compared to experiment [4], indicating the necessity of incorporating nonfactorized effects to suppress the $\Delta I = 3/2$ amplitude [5]. This implies that not all the long-distance nonfactorized contributions to hadronic matrix elements are fully accounted for by chiral loops. In short, it is not possible to reproduce A_0 and A_2 amplitudes *simultaneously* by chiral loops alone.

Even if the scale dependence of $K \to \pi\pi$ matrix elements can be furnished by meson loops, it is clear that this approach based on chiral perturbation theory is not applicable to heavy meson decays. Therefore, it is strongly desirable to describe the nonleptonic decays of kaons and heavy mesons within the same framework.

In the effective Hamiltonian approach, the renormalization scale and scheme dependence of Wilson coefficients is compensated by that of the hadronic matrix elements of four-quark operators $O(\mu)$ renormalized at the scale μ. Since there is no first-principles evaluation of $\langle O(\mu)\rangle$ except for lattice calculations, it becomes necessary to compute the vertex- and penguin-type corrections to $\langle O\rangle$ (not $\langle O(\mu)\rangle$!), which account for the scale and scheme dependence of $\langle O(\mu)\rangle$, and then apply other methods to calculate $\langle O\rangle$. The $\Delta I = 1/2$ rule

arises from the cumulative effects of the short-distance Wilson coefficients, penguin operators, final-state interactions, nonfactorized effects due to soft-gluon exchange, and radiative corrections to the matrix elements of four-quark operators. As shown in [6], the last two effects are the main ingredients for the large enhancement of A_0 with respect to A_2.

Contrary to the nonfactorized effects in charmless B decays, which are dominated by hard gluon exchange in the heavy quark limit [7] and expected to be small due to the large energy released in the decay process, the nonfactorized term in $K \to \pi\pi$ is anticipated to be large and nonperturbative in nature, namely it comes mainly from soft gluon exchange. One can use $K^+ \to \pi^+\pi^0$ to extract the nonfactorizable contributions to the hadronic matrix elements of $(V-A)(V-A)$ four-quark operators [6].

Instead of using scheme- and scale-independent effective Wilson coefficients, one can alternatively parametrize the hadronic matrix elements in terms of the bag parameters $B_i^{(0)}$ and $B_i^{(2)}$ which describe the scale and scheme dependence of hadronic matrix elements $\langle Q_i(\mu)\rangle$. These non-perturbative parameters are evaluated in the present paper. We have checked explicitly that these two seemingly different approaches yield the same results.

The prediction of ε'/ε in the standard model is often plagued by the difficulty that the result depends on the choice of the renormalization scheme. Presumably this is not an issue in the effective Wilson coefficient approach. Unfortunately, as we shall see in Sec. IV, our predictions for ε'/ε are scheme dependent for reasons not clear to us.

The present paper is organized as follows. In Sec. II we construct scheme and scale independent effective Wilson coefficients relevant to $K \to \pi\pi$ decays and direct CP violation ε'/ε. The bag parameters B_i are evaluated in Sec. III. Based on the effective Wilson coefficients or bag parameters, $K \to \pi\pi$ amplitudes and direct CP violation are calculated in Sec. IV and their results are discussed in Sec. V. Sec. VI is for the conclusion.

II. EFFECTIVE WILSON COEFFICIENTS

The effective Hamiltonian relevant to $K \to \pi\pi$ transition is

$$\mathcal{H}_{\rm eff}(\Delta S=1) = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \left(\sum_{i=1}^{10} c_i(\mu) Q_i(\mu) \right) + \text{h.c.}, \tag{2.1}$$

where

$$c_i(\mu) = z_i(\mu) + \tau y_i(\mu), \tag{2.2}$$

with $\tau = -V_{td}V_{ts}^*/(V_{ud}V_{us}^*)$, and

$$\begin{aligned}
&Q_1 = (\bar{u}d)_{V-A}(\bar{s}u)_{V-A}, && Q_2 = (\bar{u}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \\
&Q_{3(5)} = (\bar{s}d)_{V-A}\sum_q (\bar{q}q)_{V-A(V+A)}, && Q_{4(6)} = (\bar{s}_\alpha d_\beta)_{V-A}\sum_q (\bar{q}_\beta q_\alpha)_{V-A(V+A)}, \\
&Q_{7(9)} = \frac{3}{2}(\bar{s}d)_{V-A}\sum_q e_q(\bar{q}q)_{V+A(V-A)}, && Q_{8(10)} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V+A(V-A)},
\end{aligned} \tag{2.3}$$

with Q_3–Q_6 being the QCD penguin operators, Q_7–Q_{10} the electroweak penguin operators and $(\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1\gamma_\mu(1\pm\gamma_5)q_2$. The sum in Eq. (2.3) is over light flavors, $q = u, d, s$.

In the absence of first-principles calculations for hadronic matrix elements, it is customary to evaluate the matrix elements under the factorization hypothesis so that $\langle Q(\mu)\rangle$ is factorized into the product of two matrix elements of single currents, governed by decay constants and form factors. However, the naive factorized amplitude is not renormalization scale- and γ_5 scheme- independent as the scale and scheme dependences of Wilson coefficients are not compensated by that of the factorized hadronic matrix elements. In principle, the scale and scheme problems with naive factorization will not occur in the full amplitude since $\langle Q(\mu)\rangle$ involves vertex-type and penguin-type corrections to the hadronic matrix elements of the 4-quark operator renormalized at the scale μ. Schematically,

$$\begin{aligned}\text{weak decay amplitude} = \text{naive factorization} \ + \ \text{vertex}-\text{type corrections} \\ + \text{penguin}-\text{type corrections} \ + \ \text{spectator contributions} + \cdots,\end{aligned} \tag{2.4}$$

where the spectator contributions take into account the gluonic interactions between the spectator quark of the kaon and the outgoing light meson. The perturbative part of vertex-type and penguin-type corrections will render the decay amplitude scale and scheme independent. Generally speaking, the Wilson coefficient $c(\mu)$ takes into account the physics evolved from the scale M_W down to μ, while $\langle Q(\mu)\rangle$ involves evolution from μ down to the infrared scale. Formally, one can write

$$\langle Q(\mu)\rangle = g(\mu,\mu_f)\langle Q(\mu_f)\rangle, \tag{2.5}$$

where μ_f is a factorization scale, and $g(\mu,\mu_f)$ is an evolution factor running from the scale μ to μ_f which is calculable because the infrared structure of the amplitude is absorbed into $\langle Q(\mu_f)\rangle$. Writing

$$c^{\rm eff}(\mu_f) = c(\mu)g(\mu,\mu_f), \tag{2.6}$$

the effective Wilson coefficients will be scheme and μ-scale independent. Of course, it appears that the μ-scale problem with naive factorization is traded in by the μ_f-scale problem. Nevertheless, once the factorization scale at which we apply the factorization approximation to matrix elements is fixed, the physical amplitude is independent of the choice of μ. More importantly, the effective Wilson coefficients are γ_5-scheme independent. In principle, one can work with any quark configuration, on-shell or off-shell, to compute the full amplitude. Note that if external quarks are off-shell and if the off-shell quark momentum is chosen as the infrared cutoff, $g(\mu,\mu_f)$ will depend on the gauge of the gluon field [8]. But this is not a problem at all since the gauge dependence belongs to the infrared structure of the wave function. However, if factorization is applied to $\langle Q(\mu_f)\rangle$, the information of the gauge dependence characterized by the wave function will be lost. Hence, as stressed in [9,10], in order to apply factorization to matrix elements and in the meantime avoid the gauge problem connected with effective Wilson coefficients, one must work in the on-shell scheme to obtain gauge invariant and infrared finite $c_i^{\rm eff}$ and then applies factorization to $\langle Q(\mu_f)\rangle$ afterwards. Of course, physics should be μ_f independent. In the formalism of the perturbative QCD factorization theorem, the nonperturbative meson wave functions are specified with the dependence on the factorization scale μ_f [9]. These wave functions are

universal for all decay processes involving the same mesons. Hence, a consistent evaluation of hadronic matrix elements will eventually resort to the above-mentioned meson wave functions determined at the scale μ_f.

In general, the scheme- and μ-scale-independent effective Wilson coefficients have the form [11,12]:

$$c_i^{\rm eff}(\mu_f) = c_i(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{ij} c_j(\mu) + \text{ penguin}-\text{type corrections}, \tag{2.7}$$

where μ_f is the factorization scale arising from the dimensional regularization of infrared divergence [9], and the anomalous dimension matrix γ_V as well as the constant matrix $\hat{r}_V$ arise from the vertex-type corrections to four-quark operators. For kaon decays under consideration, there is no any heavy quark mass scale between m_c and m_K. Hence, the logarithmic term emerged in the vertex corrections to 4-quark operators is of the form $\ln(\mu_f/\mu)$ as shown in Eq. (2.7). We will set $\mu_f = 1$ GeV in order to have a reliable estimate of perturbative effects on effective Wilson coefficients.

It is known that the penguin operators $Q_{5,6}$ do not induce $K^0 \to \pi\pi$ directly, but their Fierz transformations via $(V-A)(V+A) \to -2(S+P)(S-P)$ do make contributions. Applying equations of motion, $\langle Q_{5,6}(\mu)\rangle$ are proportional to $m_K^2/[m_s(\mu)+m_q(\mu)]$ with $q=u$ or d. This means that, contrary to current$\times$current operators, the matrix elements $\langle Q_{5,6}(\mu)\rangle$ for $K-\pi\pi$ transition under the vacuum insertion approximation *do* exhibit a μ dependence governed by light quark masses. The μ dependence of the Wilson coefficients $c_{5,6}(\mu)$ is essentially compensated by that of light quark masses (the cancellation becomes exact in the large-N_c limit). Of course, the near cancellation of μ dependence does not imply that factorization works for the matrix elements of density$\times$density operators since the scheme dependence of $c_{5,6}(\mu)$ still does not get compensation. It is thus advantageous to apply the aforementioned effective Wilson coefficients to avoid the scheme problem caused by factorization. And in the meantime, the μ_f dependence of $c_{5,6}^{\rm eff}(\mu_f)$ is largely canceled by that of quark masses entering the matrix elements $\langle Q_{5,6}(\mu_f)\rangle$.

To proceed, we note that the renormalization-scale and -scheme independent effective Wilson coefficient functions $\tilde{z}_i^{\rm eff}$ are given by (for details, see [9,10]):

$$\begin{aligned}
\tilde{z}_1^{\rm eff} &= z_1(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{1i} z_i(\mu), \\
\tilde{z}_2^{\rm eff} &= z_2(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{2i} z_i(\mu), \\
\tilde{z}_3^{\rm eff} &= z_3(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{3i} z_i(\mu) - \frac{\alpha_s}{24\pi}(C_t + C_p), \\
\tilde{z}_4^{\rm eff} &= z_4(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{4i} z_i(\mu) + \frac{\alpha_s}{8\pi}(C_t + C_p), \\
\tilde{z}_5^{\rm eff} &= z_5(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{5i} z_i(\mu) - \frac{\alpha_s}{24\pi}(C_t + C_p), \\
\tilde{z}_6^{\rm eff} &= z_6(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{6i} z_i(\mu) + \frac{\alpha_s}{8\pi}(C_t + C_p),
\end{aligned}$$

$$
\begin{aligned}
\tilde{z}_7^{\rm eff} &= z_7(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{7i} z_i(\mu) + \frac{\alpha}{8\pi} C_e, \\
\tilde{z}_8^{\rm eff} &= z_8(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{8i} z_i(\mu), \\
\tilde{z}_9^{\rm eff} &= z_9(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{9i} z_i(\mu) + \frac{\alpha}{8\pi} C_e, \\
\tilde{z}_{10}^{\rm eff} &= z_{10}(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{10i} z_i(\mu).
\end{aligned} \tag{2.8}
$$

Likewise, the effective Wilson coefficients $\tilde{y}_i^{\rm eff}$ have similar expressions in terms of $y_i(\mu)$ except that $y_1 = y_2 = 0$. The reason why we call the l.h.s. of Eq. (2.8) as $\tilde{z}_i^{\rm eff}$ rather than $z_i^{\rm eff}$ will become shortly. In Eq. (2.8) the superscript T denotes a transpose of the matrix, the anomalous dimension matrix γ_V as well as the constant matrix $\hat{r}_V$ arise from the vertex corrections to the operators $Q_1 - Q_{10}$, C_t, C_p and C_e from the QCD penguin-type diagrams of the operators $Q_{1,2}$, the QCD penguin-type diagrams of the operators $Q_3 - Q_6$, and the electroweak penguin-type diagram of $Q_{1,2}$, respectively:

$$
\begin{aligned}
C_t &= \left(\frac{2}{3}\kappa - G(m_u,k,\mu)\right) z_1, \\
C_p &= \left(\frac{4}{3}\kappa - G(m_s,k,\mu) - G(m_d,k,\mu)\right) z_3 - \sum_{i=u,d,s} G(m_i)(z_4+z_6), \\
C_e &= \frac{8}{9}\left(\frac{2}{3}\kappa - G(m_u,k,\mu)\right)(z_1+3z_2),
\end{aligned} \tag{2.9}
$$

where κ is a parameter characterizing the γ_5-scheme dependence in dimensional regularization, i.e.,

$$
\kappa = \begin{cases} 1 & \text{NDR}, \\ 0 & \text{HV}, \end{cases} \tag{2.10}
$$

in the naive dimensional regularization (NDR) and 't Hooft-Veltman (HV) schemes for γ_5, and the function $G(m,k,\mu)$ is given by

$$
G(m,k,\mu) = -4\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2 - k^2 x(1-x)}{\mu^2}\right), \tag{2.11}
$$

with k^2 being the momentum squared carried by the virtual gluon. The explicit expression for γ_V is given in [11]. For reader's convenience, we list here the constant matrix $\hat{r}_V$ [10,6]:

$$
\hat{r}_V^{\rm NDR} = \begin{pmatrix}
3 & -9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3 & -9 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -3 & 17 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -3 & 17 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9 & 3
\end{pmatrix} \tag{2.12}
$$

in the NDR scheme, and

$$\hat{r}_V^{\rm HV} = \begin{pmatrix} \frac{7}{3} & -7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -7 & \frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{7}{3} & -7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -7 & \frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{47}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{47}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{3} & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & \frac{7}{3} \end{pmatrix} \tag{2.13}$$

in the HV scheme. Note that the 66 and 88 entries of $\hat{r}_V$ given in [10] are erroneous and have been corrected in [6] and [13].

The results of a direct calculation of $\tilde{z}_i^{\rm eff}$ and $\tilde{y}_i^{\rm eff}$ in NDR and HV schemes using Eq. (2.8) are displayed in Table I. Formally, the effective Wilson coefficients are scale and scheme independent up to the order α_s. This implies that the scheme independence of $z_i^{\rm eff}$ requires that the Wilson coefficients $z_i(\mu)$ appearing in the vertex-type corrections and in C_t, C_p and C_e be replaced by the lowest-order (LO) ones $z_i^{\rm LO}$, while z_1 appearing in C_t be the next-to-leading order (NLO) one, and likewise for $y_i^{\rm eff}$. Therefore, we define the effective Wilson coefficients $z_i^{\rm eff}$ and $y_i^{\rm eff}$ similar to Eq. (2.8) except for the above-mentioned replacement, for example,

$$z_1^{\rm eff} = z_1(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^T \ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{1i} z_i^{\rm LO}(\mu). \tag{2.14}$$

From Table I we see that the scale independence of $z_i^{\rm eff}$, which is good to the accuracy of the third digit, is significantly better than $\tilde{z}_i^{\rm eff}$; for example, the values of $z_{4,6}$(NDR) and $z_{4,6}$(HV) to NLO are quite different, but their effective Wilson coefficients $z_{4,6}^{\rm eff}$ are obviously scheme independent. Note that $z_3^{\rm eff},\cdots,z_6^{\rm eff}$ are enhanced relative to their NLO values by about three times as they receive large corrections proportional to $\alpha_s C_t$. By contrast, the scheme independence of $y_i^{\rm eff}$ is not as good as $z_i^{\rm eff}$. In particular, $y_6^{\rm eff}$, which plays an important role in ε'/ε, shows a slight scheme dependence for reasons not clear to us.

The effective Wilson coefficients appear in the factorizable decay amplitudes in the combinations $a_{2i} = z_{2i}^{\rm eff} + \frac{1}{N_c} z_{2i-1}^{\rm eff}$ and $a_{2i-1} = z_{2i-1}^{\rm eff} + \frac{1}{N_c} z_{2i}^{\rm eff}$ $(i = 1,\cdots,5)$. For $K \to \pi\pi$ decays, nonfactorizable effects in hadronic matrix elements can be absorbed into the parameters $a_i^{\rm eff}$ [14–16]:

$$a_{2i}^{\rm eff} = z_{2i}^{\rm eff} + \left(\frac{1}{N_c} + \chi_{2i}\right) z_{2i-1}^{\rm eff}, \qquad a_{2i-1}^{\rm eff} = z_{2i-1}^{\rm eff} + \left(\frac{1}{N_c} + \chi_{2i-1}\right) z_{2i}^{\rm eff}, \tag{2.15}$$

with χ_i being the nonfactorizable terms. Likewise,

$$b_{2i}^{\rm eff} = y_{2i}^{\rm eff} + \left(\frac{1}{N_c} + \chi_{2i}\right) y_{2i-1}^{\rm eff}, \qquad b_{2i-1}^{\rm eff} = y_{2i-1}^{\rm eff} + \left(\frac{1}{N_c} + \chi_{2i-1}\right) y_{2i}^{\rm eff}. \tag{2.16}$$

TABLE I. $\Delta S=1$ Wilson coefficients at $\mu=1$ GeV for $m_t=170$ GeV and $\Lambda^{(4)}_{\overline{\rm MS}}=325$ MeV, taken from Table XVIII of [17]. Also shown are the effective Wilson coefficients $\tilde{z}_i^{\rm eff}$ and $z_i^{\rm eff}$ (see the text), $\tilde{y}_i^{\rm eff}$ and $y_i^{\rm eff}$ in NDR and HV schemes with $\mu=1$ GeV, $\mu_f=1$ GeV and $k^2=m_K^2/2$. Note that $y_1=y_2=0$.

	LO	NDR	HV	$\tilde{z}_i^{\rm eff}$(NDR)	$\tilde{z}_i^{\rm eff}$(HV)	$z_i^{\rm eff}$(NDR)	$z_i^{\rm eff}$(HV)
z_1	1.433	1.278	1.371	1.614	1.678	1.718	1.713
z_2	-0.748	-0.509	-0.640	-1.029	-1.082	-1.113	-1.110
z_3	0.004	0.013	0.007	0.039	0.034	0.032	0.032
z_4	-0.012	-0.035	-0.017	-0.080	-0.085	-0.081	-0.084
z_5	0.004	0.008	0.004	0.024	0.024	0.024	0.025
z_6	-0.013	-0.035	-0.014	-0.094	-0.086	-0.086	-0.086
z_7/α	0.008	0.011	-0.002	0.025	0.047	0.063	0.069
z_8/α	0.001	0.014	0.010	0.025	0.016	0.016	0.013
z_9/α	0.008	0.018	0.005	0.039	0.057	0.072	0.078
z_{10}/α	-0.001	-0.008	-0.010	-0.015	-0.012	-0.011	-0.012
	LO	NDR	HV	$\tilde{y}_i^{\rm eff}$(NDR)	$\tilde{y}_i^{\rm eff}$(HV)	$y_i^{\rm eff}$(NDR)	$y_i^{\rm eff}$(HV)
y_3	0.038	0.032	0.037	0.048	0.050	0.050	0.049
y_4	-0.061	-0.058	-0.061	-0.051	-0.055	-0.053	-0.053
y_5	0.013	-0.001	0.016	0.004	0.003	0.003	0.002
y_6	-0.113	-0.111	-0.097	-0.161	-0.130	-0.160	-0.138
y_7/α	0.036	-0.032	-0.030	-0.051	-0.019	-0.052	-0.028
y_8/α	0.158	0.173	0.188	0.287	0.295	0.285	0.300
y_9/α	-1.585	-1.576	-1.577	-2.013	-1.919	-2.053	-1.948
y_{10}/α	0.800	0.690	0.699	1.339	1.205	1.355	1.216

To proceed, we shall assume that nonfactorizable effects in the matrix elements of $(V-A)(V+A)$ operators differ from that of $(V-A)(V-A)$ operators; that is,

$$\chi_{LL}\equiv\chi_1=\chi_2=\chi_3=\chi_4=\chi_9=\chi_{10},$$
$$\chi_{LR}\equiv\chi_5=\chi_6=\chi_7=\chi_8, \tag{2.17}$$

and $\chi_{LR}\neq\chi_{LL}$. Theoretically, a primary reason is that the Fierz transformation of the $(V-A)(V+A)$ operators $O_{5,6,7,8}$ is quite different from that of $(V-A)(V-A)$ operators $O_{1,2,3,4}$ and $O_{9,10}$ [10]. Experimentally, we have learned from nonleptonic charmless B decays that $\chi_{LR}(B)\neq\chi_{LL}(B)$ [10,13]. As shown in [6], the nonfactorized term χ_{LL} can be extracted from $K^+\to\pi^+\pi^0$ decay to be

$$\chi_{LL}=-0.73\,. \tag{2.18}$$

Contrary to the nonfactorized effects in hadronic charmless B decays, which are dominated by hard gluon exchange in the heavy quark limit [7] and expected to be small due to the

large energy released in the decay process, the nonfactorized term in $K \to \pi\pi$ is large and nonperturbative in nature, namely it comes mainly from soft gluon exchange.

III. NON-PERTURBATIVE PARAMETERS B_i

In the literature it is often to parametrize the hadronic matrix elements in terms of the non-perturbative bag parameters $B_i^{(0)}$ and $B_i^{(2)}$ which describe the scale and scheme dependence of the hadronic matrix elements $\langle Q_i(\mu)\rangle$:

$$B_i^{(0)}(\mu) \equiv \frac{\langle Q_i(\mu)\rangle_0}{\langle Q_i\rangle_0^{\rm VIA}}, \qquad B_i^{(2)}(\mu) \equiv \frac{\langle Q_i(\mu)\rangle_2}{\langle Q_i\rangle_2^{\rm VIA}}, \tag{3.1}$$

where $\langle Q_i\rangle^{\rm VIA}$ denote the matrix elements evaluated under the vacuum insertion approximation. In order to evaluate the parameters $B_i^{(0,2)}$, as an example we consider the vertex-type and penguin-type corrections to the hadronic matrix element of the four-quark operator Q_1 in the NDR scheme [10]:

$$\begin{aligned}\langle Q_1(\mu)\rangle = &\left[1+\frac{\alpha_s}{4\pi}\left(-2\ln\frac{\mu_f}{\mu}+3\right)\right]\langle Q_1(\mu_f)\rangle + \frac{\alpha_s}{4\pi}\left(6\ln\frac{\mu_f}{\mu}-9\right)\langle Q_2(\mu_f)\rangle \\ &-\frac{\alpha_s}{8\pi}\left(G(m_u,k,\mu)-\frac{2}{3}\right)\langle P(\mu_f)\rangle - \frac{\alpha}{9\pi}\left(G(m_u,k,\mu)-\frac{2}{3}\right)\langle Q_7(\mu_f)+Q_9(\mu_f)\rangle,\end{aligned} \tag{3.2}$$

where

$$P = -\frac{1}{3}Q_3 + Q_4 - \frac{1}{3}Q_5 + Q_6. \tag{3.3}$$

The parameters $B_1^{(0,2)}$ are then determined from Eq. (3.2). In general, the hadronic parameters $B_i^{(0,2)}(\mu,\mu_f)$ have the expressions:*

$$\begin{aligned}B_1^{(0)} = &\left\{\langle Q_1\rangle_0 + \frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat r_V\right)_{1i}\langle Q_i\rangle_0 - \frac{\alpha_s}{8\pi}\left(G(m_u)-\frac{2}{3}\kappa\right)\langle P\rangle_0\right. \\ &\left.-\frac{\alpha}{9\pi}\left(G(m_u)-\frac{2}{3}\kappa\right)\langle Q_7+Q_9\rangle_0\right\}\Big/\langle Q_1\rangle_0^{\rm VIA}, \\ B_1^{(2)} = &\left\{\langle Q_1\rangle_2 + \frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat r_V\right)_{1i}\langle Q_i\rangle_2\right. \\ &\left.-\frac{\alpha}{9\pi}\left(G(m_u)-\frac{2}{3}\kappa\right)\langle Q_7+Q_9\rangle_2\right\}\Big/\langle Q_1\rangle_2^{\rm VIA}, \\ B_2^{(0,2)} = &\left\{\langle Q_2\rangle_{0,2} + \frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat r_V\right)_{2i}\langle Q_i\rangle_{0,2}\right.\end{aligned}$$

---

*Note that our convention for Q_1, Q_2 (and hence B_1, B_2) differs from that in [18,17] where the labels 1 and 2 are interchanged.

$$
\begin{aligned}
& -\frac{\alpha}{3\pi}\left(G(m_u)-\frac{2}{3}\kappa\right)\langle Q_7+Q_9\rangle_{0,2}\Big\}\Big/\langle Q_2\rangle_{0,2}^{\rm VIA}, \\
B_3^{(0)} &= \left\{\langle Q_3\rangle_0+\frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat{r}_V\right)_{3i}\langle Q_i\rangle_0\right. \\
& \left.-\frac{\alpha_s}{4\pi}\left(G(m_d)+G(m_s)-\frac{4}{3}\kappa\right)\langle P\rangle_0\right\}\Big/\langle Q_3\rangle_0^{\rm VIA}, \\
B_4^{(0)} &= \left\{\langle Q_4\rangle_0+\frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat{r}_V\right)_{4i}\langle Q_i\rangle_0\right. \\
& \left.-\frac{\alpha_s}{4\pi}\left[G(m_u)+G(m_d)+G(m_s)\right]\langle P\rangle_0\right\}\Big/\langle Q_4\rangle_0^{\rm VIA}, \\
B_5^{(0)} &= \left\{\langle Q_5\rangle_0+\frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat{r}_V\right)_{5i}\langle Q_i\rangle_0\right\}\Big/\langle Q_5\rangle_0^{\rm VIA}, \\
B_6^{(0)} &= \left\{\langle Q_6\rangle_0+\frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat{r}_V\right)_{6i}\langle Q_i\rangle_0\right. \\
& \left.-\frac{\alpha_s}{4\pi}\left[G(m_u)+G(m_d)+G(m_s)\right]\langle P\rangle_0\right\}\Big/\langle Q_6\rangle_0^{\rm VIA}, \\
B_7^{(0,2)} &= \left\{\langle Q_7\rangle_{0,2}+\frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat{r}_V\right)_{7i}\langle Q_i\rangle_{0,2}\right\}\Big/\langle Q_7\rangle_{0,2}^{\rm VIA}, \\
B_8^{(0,2)} &= \left\{\langle Q_8\rangle_{0,2}+\frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat{r}_V\right)_{8i}\langle Q_i\rangle_{0,2}\right\}\Big/\langle Q_8\rangle_{0,2}^{\rm VIA}, \\
B_9^{(0,2)} &= \left\{\langle Q_9\rangle_{0,2}+\frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat{r}_V\right)_{9i}\langle Q_i\rangle_{0,2}\right\}\Big/\langle Q_9\rangle_{0,2}^{\rm VIA}, \\
B_{10}^{(0,2)} &= \left\{\langle Q_{10}\rangle_{0,2}+\frac{\alpha_s}{4\pi}\left(\gamma_V\ln\frac{\mu_f}{\mu}+\hat{r}_V\right)_{10i}\langle Q_i\rangle_{0,2}\right\}\Big/\langle Q_{10}\rangle_{0,2}^{\rm VIA}.
\end{aligned}
\tag{3.4}
$$

For simplicity we have dropped the parameters k and μ in the argument of the function G. Note that the effective Wilson coefficient $\tilde{z}_i^{\rm eff}$ is in general not equal to $z_i(\mu)B_i(\mu)$, but the physical amplitude in terms of $\tilde{z}_i^{\rm eff}$ or $z_i(\mu)B_i(\mu)$ is the same.

The $K\to\pi\pi$ matrix elements under the vacuum insertion approximation read (see e.g., [18])

$$
\begin{aligned}
\langle Q_1\rangle_0 &= \frac{1}{3}X\left(2-\frac{1}{N_c}\right), & \langle Q_1\rangle_2 &= \frac{\sqrt{2}}{3}X\left(1+\frac{1}{N_c}\right), \\
\langle Q_2\rangle_0 &= \frac{1}{3}X\left(-1+\frac{2}{N_c}\right), & \langle Q_2\rangle_2 &= \frac{\sqrt{2}}{3}X\left(1+\frac{1}{N_c}\right), \\
\langle Q_3\rangle_0 &= \frac{1}{N_c}X, & \langle Q_4\rangle_0 &= X, \\
\langle Q_5\rangle_0 &= -\frac{4}{N_c}\sqrt{\frac{3}{2}}\,v^2(f_K-f_\pi), & \langle Q_6\rangle_0 &= -4\sqrt{\frac{3}{2}}\,v^2(f_K-f_\pi), \\
\langle Q_7\rangle_0 &= \frac{\sqrt{6}}{N_c}f_Kv^2+\frac{1}{2}X, & \langle Q_7\rangle_2 &= \frac{\sqrt{3}}{N_c}f_\pi v^2-\frac{1}{\sqrt{2}}X,
\end{aligned}
$$

$$\langle Q_8\rangle_0 = \sqrt{6} f_K v^2 + \frac{1}{2N_c} X, \qquad \langle Q_8\rangle_2 = \sqrt{3} f_\pi v^2 - \frac{1}{N_c\sqrt{2}} X,$$
$$\langle Q_9\rangle_0 = -\frac{1}{2} X \left(1 - \frac{1}{N_c}\right), \qquad \langle Q_9\rangle_2 = -\frac{1}{\sqrt{2}} X \left(1 + \frac{1}{N_c}\right),$$
$$\langle Q_{10}\rangle_0 = \frac{1}{2} X \left(1 - \frac{1}{N_c}\right), \qquad \langle O_{10}\rangle_2 = \frac{1}{\sqrt{2}} X \left(1 + \frac{1}{N_c}\right), \tag{3.5}$$

where $X = \sqrt{3/2}\, f_\pi (m_K^2 - m_\pi^2)$, and

$$v = \frac{m_{\pi^\pm}^2}{m_u + m_d} = \frac{m_{K^0}^2}{m_d + m_s} = \frac{m_K^2 - m_\pi^2}{m_s - m_u} \tag{3.6}$$

characterizes the quark-order parameter $\langle \bar{q} q\rangle$ which breaks chiral symmetry spontaneously.

To evaluate $B_i^{(0,2)}$ we need to take into account nonfactorized effects on hadronic matrix elements. As discussed in Sec. II, this amounts to replacing $1/N_c$ by $1/N_c + \chi_{LL}$ for $(V-A)(V-A)$ quark operators and by $1/N_c + \chi_{LR}$ for $(V-A)(V+A)$ operators. For example,

$$\frac{\langle Q_1\rangle_0}{\langle Q_1\rangle_0^{\rm VIA}} = 1 - \frac{3}{5}\chi_{LL}, \qquad \frac{\langle Q_2\rangle_0}{\langle Q_2\rangle_0^{\rm VIA}} = 1 - 6\chi_{LL}, \qquad \frac{\langle Q_5\rangle_0}{\langle Q_5\rangle_0^{\rm VIA}} = 1 + 3\chi_{LR}. \tag{3.7}$$

Although the nonfactorized term χ_{LL} is fixed by the measurement of $K^+ \to \pi^+\pi^0$ to be -0.73 [6], no constraint on χ_{LR} can be extracted from $K^0 \to \pi\pi$. Nevertheless, we learned from hadronic charmless B decays that $\chi_{LR} \neq \chi_{LL}$ [10]. As shown in Fig. 1, the parameters B_5, $B_7^{(0)}$ and $B_7^{(2)}$ are quite sensitive to χ_{LR}, whereas B_6, $B_7^{(0)}$ and $B_7^{(2)}$ stay stable. Lattice calculations suggest that $B_5 \simeq B_6 = 1.0 \pm 0.2$ and $B_7^{(2)} = 0.6 \pm 0.1$ at $\mu = 2$ GeV [19]. The lattice results roughly imply the constraint $-0.45 < \chi_{LR} < 0$. We shall see in Sec. IV that the A_0/A_2 ratio and ε'/ε are not very sensitive to the variation of χ_{LR}. Using $m_u = 3.5$ MeV, $m_d = 7.0$ MeV, $m_s = 140$ MeV at $\mu = 1$ GeV and $\chi_{LR} = -0.1$, the numerical values of $B_i^{(0,2)}$ are listed in Tables II and III.

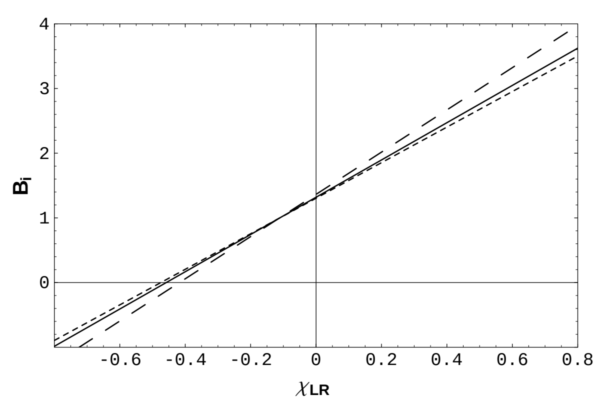

FIG. 1. The bag parameters B_5 (solid line), $B_7^{(0)}$ (dotted line) and $B_7^{(2)}$ (dashed line) evaluated in the NDR scheme versus χ_{LR} for $m_s(1\,\text{GeV}) = 140$ MeV.

TABLE II. Numerical values of the non-perturbative bag parameters $B_i^{(0)}$ at $\mu = 1$ GeV in NDR and HV schemes for $m_s(1\,\mathrm{GeV}) = 140$ MeV, $\mu_f = 1$ GeV and $k^2 = m_K^2/2$, $\chi_{LL} = -0.73$ and $\chi_{LR} = -0.1$. The results for $B_i^{(0)}$ in the absence of nonfactorized contributions (i.e. $\chi_{LL} = \chi_{LR} = 0$) are shown in parentheses.

	$B_1^{(0)}$	$B_2^{(0)}$	$B_3^{(0)}$	$B_4^{(0)}$	$B_5^{(0)}$	$B_6^{(0)}$	$B_7^{(0)}$	$B_8^{(0)}$	$B_9^{(0)}$	$B_{10}^{(0)}$
NDR	2.5(1.8)	8.7(3.1)	-0.1(2.8)	2.7(2.7)	1.0(1.3)	1.5(1.5)	1.0(1.3)	1.6(1.6)	3.1(1.5)	3.1(1.5)
HV	2.6(2.1)	8.0(2.7)	0.9(3.8)	2.6(2.7)	1.7(2.0)	1.5(1.5)	1.7(1.9)	1.6(1.6)	2.9(1.4)	2.9(1.4)

TABLE III. Same as Table II except for $B_i^{(2)}$.

	$B_1^{(2)}$	$B_2^{(2)}$	$B_7^{(2)}$	$B_8^{(2)}$	$B_9^{(2)}$	$B_{10}^{(2)}$
NDR	0.34(0.75)	0.34(0.74)	1.0(1.4)	1.6(1.6)	0.35(0.76)	0.35(0.76)
HV	0.37(0.81)	0.36(0.79)	1.8(2.1)	1.6(1.6)	0.37(0.81)	0.37(0.81)

IV. $K \to \pi\pi$ ISOSPIN AMPLITUDES AND ε'/ε

In terms of the effective Wilson coefficients defined in Sec. II, the CP-even $\Delta I = 1/2$ and $\Delta I = 3/2$ $K \to \pi\pi$ amplitudes have the form [6]:

$$\begin{aligned}
\mathrm{Re}A_0 &= \frac{G_F}{\sqrt{2}}\,\frac{\mathrm{Re}(V_{ud}V_{us}^*)}{\cos\delta_0}\Big\{\Big[\frac{2}{3}a_1 - \frac{1}{3}a_2 + a_4 + \frac{1}{2}(a_7 - a_9 + a_{10})\Big]X \\
&\qquad -2\sqrt{6}\,v^2(f_K - f_\pi)a_6 + \sqrt{6}\,v^2 f_K a_8\Big\}, \\
\mathrm{Re}A_2 &= \frac{G_F}{\sqrt{2}}\,\frac{\mathrm{Re}(V_{ud}V_{us}^*)}{\cos\delta_2}\,\frac{1}{1-\Omega_{\mathrm{IB}}}\Big\{\Big[a_1 + a_2 + \frac{3}{2}(-a_7 + a_9 + a_{10})\Big]\frac{\sqrt{2}}{3}X + \sqrt{3}\,f_\pi v^2 a_8\Big\},
\end{aligned} \tag{4.1}$$

where $\Omega_{\mathrm{IB}} \equiv A_2^{\mathrm{IB}}/A_2$ describes the isospin breaking contribution to $K^+ \to \pi^+\pi^0$ due to the $\pi - \eta - \eta'$ mixing, and δ_0 as well as δ_2 are S-wave $\pi\pi$ scattering isospin phase shifts. For simplicity, we have dropped the superscript "eff" of a_i in Eq. (4.1).

The direct CP-violation parameter ε'/ε given by the general expression

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}|\varepsilon|}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right) \tag{4.2}$$

can be recast in the form

$$\begin{aligned}
\frac{\varepsilon'}{\varepsilon} = \frac{G_F\omega}{2|\varepsilon|\mathrm{Re}A_0}\mathrm{Im}(V_{td}V_{ts}^*)\Big\{&\frac{1}{\cos\delta_0}\Big[(b_4 + \frac{1}{2}b_7 - \frac{1}{2}b_9 + \frac{1}{2}b_{10})X \\
&-2\sqrt{6}\,v^2(f_K - f_\pi)b_6 + \sqrt{6}\,v^2 f_K b_8\Big](1-\Omega_{\mathrm{IB}}) \\
&-\frac{1}{\omega}\frac{1}{\cos\delta_2}\Big[(-b_7 + b_9 + b_{10})X/\sqrt{2} + \sqrt{3}\,v^2 f_\pi b_8\Big]\Big\},
\end{aligned} \tag{4.3}$$

where $\omega \equiv A_2/A_0 = 1/22.2$.

Alternatively, the $K \to \pi\pi$ amplitudes and direct CP violation can be expressed in terms of the non-perturbative parameters $B_i^{(0,2)}$:

$$\begin{aligned}
\mathrm{Re}A_0 &= \frac{G_F}{\sqrt{2}} \frac{\mathrm{Re}(V_{ud}V_{us}^*)}{\cos\delta_0} \sum_{i=1}^{10} z_i B_i^{(0)} \langle Q_i \rangle_0^{\mathrm{VIA}}, \\
\mathrm{Re}A_2 &= \frac{G_F}{\sqrt{2}} \frac{\mathrm{Re}(V_{ud}V_{us}^*)}{\cos\delta_2} \frac{1}{1-\Omega_{\mathrm{IB}}} \sum_{i=1}^{10} z_i B_i^{(2)} \langle Q_i \rangle_2^{\mathrm{VIA}},
\end{aligned} \tag{4.4}$$

and

$$\begin{aligned}
\frac{\varepsilon'}{\varepsilon} = \frac{G_F\omega}{2|\varepsilon|\mathrm{Re}A_0} \mathrm{Im}(V_{td}V_{ts}^*) \Bigg\{ & \frac{1}{\cos\delta_0} \sum_{i=3}^{10} y_i B_i^{(0)} \langle Q_i \rangle_0^{\mathrm{VIA}} (1-\Omega_{\mathrm{IB}}) \\
& - \frac{1}{\omega} \frac{1}{\cos\delta_2} \sum_{i=3}^{10} y_i B_i^{(2)} \langle Q_i \rangle_2^{\mathrm{VIA}} \Bigg\}.
\end{aligned} \tag{4.5}$$

We have checked explicitly that the numerical values of $\Delta I = 1/2,\ 3/2$ amplitudes and ε'/ε obtained using the parameters $B_i^{(0,2)}$ given in Tables II and III are the same as that calculated using the effective Wilson coefficients $\tilde{z}_i^{\mathrm{eff}}$ and $\tilde{y}_i^{\mathrm{eff}}$ shown in Table I, as it should be. Since z_i^{eff} and y_i^{eff} show a better scale independence than $\tilde{z}_i^{\mathrm{eff}}$ and $\tilde{y}_i^{\mathrm{eff}}$, we will use the former set of effective Wilson coefficients in ensuing calculations.†

Using $\Omega_{\mathrm{IB}} = 0.25 \pm 0.02$ [6], $\delta_0 = (34.2 \pm 2.2)^\circ$, $\delta_2 = -(6.9 \pm 0.2)^\circ$ [20], $\chi_{LL} = -0.73$ and $\chi_{LR} = -0.1$, we plot in Fig. 2 the ratio A_0/A_2 as a function of m_s at the renormalization scale $\mu = 1$ GeV. Specifically, we obtain

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = \begin{cases} 15.2 & \text{at } m_s\,(1\,\mathrm{GeV}) = 125 \text{ MeV}, \\ 13.6 & \text{at } m_s\,(1\,\mathrm{GeV}) = 150 \text{ MeV}, \\ 12.7 & \text{at } m_s\,(1\,\mathrm{GeV}) = 175 \text{ MeV}. \end{cases} \tag{4.6}$$

It is clear that the strange quark mass is favored to be smaller and that the prediction is renormalization scheme independent, as it should be. In Fig. 3 we study the dependence of A_0/A_2 on the unknown nonfactorized term χ_{LR}. It turns out that the ratio decreases slowly with χ_{LR}, but it stays stable within the allowed region $-0.45 < \chi_{LR} < 0$.

For direct CP violation, we find for $\mathrm{Im}(V_{td}V_{ts}^*) = 1.29 \times 10^{-4}$ (see Fig. 4)

$$\frac{\varepsilon'}{\varepsilon} = \begin{cases} 1.56\ (1.02) \times 10^{-3} & \text{at } m_s\,(1\,\mathrm{GeV}) = 125 \text{ MeV}, \\ 1.07\ (0.70) \times 10^{-3} & \text{at } m_s\,(1\,\mathrm{GeV}) = 150 \text{ MeV}, \\ 0.78\ (0.51) \times 10^{-3} & \text{at } m_s\,(1\,\mathrm{GeV}) = 175 \text{ MeV}, \end{cases} \tag{4.7}$$

---

†The results of A_0/A_2 and ε'/ε calculated using the bag parameters $B_i^{(0,2)}$ and effective Wilson coefficients z_i^{eff} and y_i^{eff} are numerically very similar except for ε'/ε in the HV scheme which is slightly small in terms of B-parameters by around 15% compared to that evaluated in terms of y_i^{eff}. For example, Eq. (4.5) leads to $\varepsilon'/\varepsilon = 0.59 \times 10^{-3}$ at $m_s(1\,\mathrm{GeV}) = 150$ MeV in the HV scheme, while Eq. (4.1) yields $\varepsilon'/\varepsilon = 0.70 \times 10^{-3}$ [see Eq. (4.7)] in the same scheme.

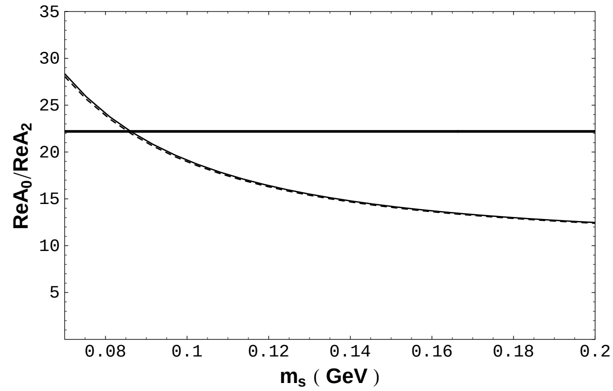

FIG. 2. The ratio of $\mathrm{Re}A_0/\mathrm{Re}A_2$ versus m_s (in units of GeV) at the renormalization scale $\mu = 1$ GeV, where the solid (dotted) curve is calculated in the NDR (HV) scheme and use of $\chi_{LR} = -0.1$ has been made. The solid thick line is the experimental value for $\mathrm{Re}A_0/\mathrm{Re}A_2$.

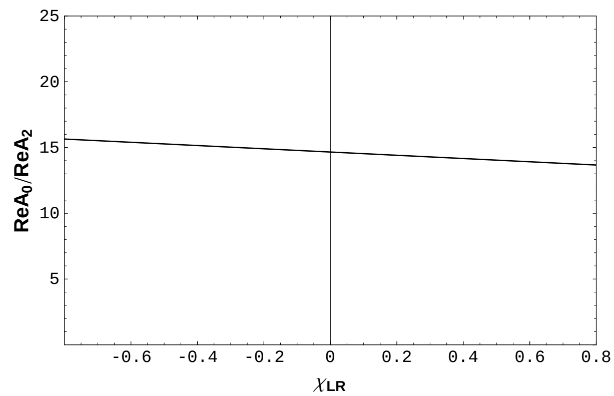

FIG. 3. The ratio of $\mathrm{Re}A_0/\mathrm{Re}A_2$ versus χ_{LR} for $m_s(1\,\mathrm{GeV}) = 140$ MeV.

in the NDR scheme, where the calculations in the HV scheme are shown in parentheses. Experimentally, the world average including NA31 [21], E731 [22], KTeV [23] and NA48 [24] results is

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (2.13 \pm 0.46) \times 10^{-3}. \tag{4.8}$$

V. DISCUSSIONS

A. Bag parameters B_i

In Sec. III we have computed the non-perturbative parameters $B_i^{(0,2)}$ at $\mu = 1$ GeV in NDR and HV schemes. Our results $B_{1,2}^{(2)} = 0.34$ and $B_{9,10}^{(2)} = 0.35$ in the NDR scheme are smaller than the value 0.48 quoted in [18] for $\mu = 1.3$ GeV. This is because we have taken into account isospin breaking contributions to the $\Delta I = 3/2$ amplitude so that $\mathrm{Re}A_2$ is enhanced by a factor of $1/(1 - \Omega_{\mathrm{IB}})$ [see Eq. (4.1)]. Consequently, it is necessary to impose large nonfactorized effects and hence small $B_{1,2}^{(2)}$ to suppress A_2. We observe from Tables II

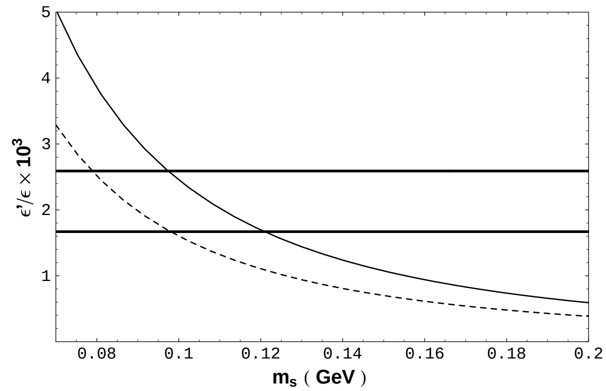

FIG. 4. Direct CP violation ε'/ε versus m_s (in units of GeV) at the renormalization scale $\mu = 1$ GeV, where the solid (dotted) curve is calculated in the NDR (HV) scheme and use of $\mathrm{Im}(V_{td}V_{ts}^*) = 1.29 \times 10^{-4}$ and $\chi_{LR} = -0.1$ has been made. The solid thick lines are the world average value for ε'/ε with one sigma errors.

and III that the parameter $B_2^{(0)}$ has the largest deviation from unity; it is equal to 8.7 in the NDR scheme. This is mainly because the ratio $\langle Q_2\rangle_0/\langle Q_2\rangle_0^{\rm VIA} = 1 - 6\chi_{LL}$ [cf. Eq. (3.7)] is greatly enhanced by the nonfactorized effect, recalling that $\chi_{LL} = -0.73$. Note that our results for $B_{1,2}^{(0)}$ are close to that obtained in the chiral quark model [5].

There exist some nonperturbative calculations for $B_6^{(0)}$ and $B_{7,8}^{(2)}$. Among them, lattice calculations are carried out at $\mu = 2$ GeV and in the NDR scheme. However, the lattice results for $B_{7,8}^{(2)}$ are much more reliable and solid than $B_6^{(0)}$. Most approaches find $B_8^{(2)}$ below unity and $B_8^{(2)} < B_6^{(0)}$, while we obtain $B_8^{(2)} = 1.6$ and $B_8^{(2)} \sim B_6^{(0)}$. Since the radiative correction $(\alpha_s/4\pi)[(\hat{r})_{87}\langle Q_7\rangle_2 + (\hat{r})_{88}\langle Q_8\rangle_2]/\langle Q_8\rangle_2^{\rm VIA}$ is positive in our case [see Eq. (3.4)], it is not possible to push $B_8^{(2)}$ down below unity.

As for the scheme dependence of B_i, it was argued in [25] that $B_6^{(0)}(\mathrm{HV}) \approx 1.2\, B_6^{(0)}(\mathrm{NDR})$ and $B_8^{(2)}(\mathrm{HV}) \approx 1.2\, B_8^{(2)}(\mathrm{NDR})$, whereas we find a very weak scheme dependence for $B_6^{(0)}$ and $B_8^{(2)}$ but strong dependence for $B_5^{(0)}$ and $B_7^{(0,2)}$: $B_5^{(0)}(\mathrm{HV}) = 1.7\, B_5^{(0)}(\mathrm{NDR})$, $B_7^{(0)}(\mathrm{HV}) = 1.7\, B_7^{(0)}(\mathrm{NDR})$ and $B_7^{(2)}(\mathrm{HV}) = 1.8\, B_7^{(2)}(\mathrm{NDR})$. We have also studied the m_s dependence of B-parameters and found that only $B_{1,3,4}^{(0)}$ exhibit a significant m_s dependence, while the other B-parameters are nearly m_s independent. For example, we obtain $B_1^{(0)} = 3.2$ if $m_s(1\,\mathrm{GeV}) = 100$ MeV and $B_1^{(0)} = 2.5$ if $m_s(1\,\mathrm{GeV}) = 140$ MeV (see Table II). Note that some other models predict a different m_s behavior for B-parameters. For example, $B_6^{(0)}$ is proportional to m_s in the chiral quark model [5].

B. $K \to \pi\pi$ amplitudes

From Fig. 2 or Eq. (4.6) we see that about (60-70)% of ReA_0 amplitude is accounted for in the present approach if $m_s(1\,\mathrm{GeV})$ lies in the range (125-175) MeV. Specifically, Q_1, Q_2 and penguin operators explain 66%, 18% and 14%, respectively, of the A_0 amplitude for $m_s(1\,\mathrm{GeV}) = 150$ MeV. Hence, tree-level current$\times$current operators account for around 85% of ReA_0. However, contrary to [3], we find that penguin-like diagrams induced by Q_1, i.e.,

the penguin operators in Eq. (3.2), contributes only about 15% to ReA_0. As conjectured in [6], the W-exchange mechanism could provide an additional important enhancement of the A_0 amplitude. Since the W-exchange amplitude in charmed meson decay is comparable to the internal W-emission one [26], it is conceivable that in kaon physics the long-distance contribution to W-exchange is as important as the external W-emission amplitude.

It is instructive to see how the predictions of ReA_0 and ReA_2 amplitudes and the $\Delta I = 1/2$ rule progress at various stages. In the absence of QCD corrections, we have $a_2 = \frac{1}{3}a_1$ and $a_3 = a_4 \cdots = a_{10} = 0$ under the vacuum insertion approximation. It follows from Eq. (4.1) that [1]

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = \frac{5}{4\sqrt{2}} = 0.9 \quad \text{(in absence of QCD corrections)}. \tag{5.1}$$

With the inclusion of lowest-order short-distance QCD corrections to the Wilson coefficients z_1 and z_2 evaluated at $\mu = 1$ GeV, A_0/A_2 is enhanced from the value of 0.9 to 2.0, and it becomes 2.3 if $m_s(1\,\mathrm{GeV}) = 150$ MeV and QCD penguin as well as electroweak penguin effects are included. This ratio is suppressed to 1.7 with the inclusion of the isospin-breaking effect, but it is increased again to the value of 2.0 in the presence of final-state interactions with $\delta_0 = 34.2°$ and $\delta_2 = -6.9°$. At this point, we have Re$A_0 = 7.7 \times 10^{-8}\,$GeV and Re$A_2 = 3.8 \times 10^{-8}\,$GeV. Comparing with the experimental values

$$\mathrm{Re}\,A_0 = 3.323 \times 10^{-7}\,\mathrm{GeV}, \qquad \mathrm{Re}\,A_2 = 1.497 \times 10^{-8}\,\mathrm{GeV}, \tag{5.2}$$

we see that the conventional calculation based on the effective Hamiltonian and naive factorization predicts a too small $\Delta I = 1/2$ amplitude by a factor of 4.3 and a too large $\Delta I = 3/2$ amplitude by a factor of 2.5 . In short, it is a long way to go to achieve the $\Delta I = 1/2$ rule within the conventional approach.

Replacing $c_i^{\mathrm{LO}}(\mu)$ by the effective Wilson coefficients c_i^{eff}, or equivalently replacing the LO Wilson coefficients by the NLO ones and including vertex-like and penguin-like corrections to four-quark operators, we obtain Re$A_0 = 1.37 \times 10^{-7}\,$GeV and Re$A_2 = 3.3 \times 10^{-8}\,$GeV and Re$A_0$/Re$A_2 = 4.2$. Finally, the inclusion of nonfactorized effects on hadronic matrix elements will enhance ReA_0/ReA_2 to the value of 13.8 with Re$A_0 = 2.07 \times 10^{-8}\,$GeV and Re$A_2 = 1.50 \times 10^{-8}\,$GeV. To summarize, the enhancement of the ratio ReA_0/ReA_2 is due to the cumulative effects of the short-distance Wilson coefficients, penguin operators, final-state interactions, nonfactorized effects due to soft-gluon exchange, and radiative corrections to the matrix elements of four-quark operators. Among them, the last two effects, which are usually not addressed in previous studies (in particular, the last one), play an essential role for explaining the bulk of the $\Delta I = 1/2$ rule.

C. Direct CP violation ε'/ε

From Fig. 4 or Eq. (4.7) we observe that, contrary to the case of A_0/A_2, the prediction of ε'/ε shows some scale dependence (see Fig. 4); roughly speaking, $(\varepsilon'/\varepsilon)_{\mathrm{NDR}} \approx 1.5\,(\varepsilon'/\varepsilon)_{\mathrm{HV}}$. To understand this, we note that ε'/ε is dominated by b_6 and b_8 terms (or y_6^{eff} and y_8^{eff}) or

equivalently by the hadronic parameters $B_6^{(0)}$ and $B_8^{(2)}$ (see Eqs. (4.2), (4.5) and Tables I-III). Moreover, direct CP violation involves a large cancellation between the dominant $y_6^{\rm eff}$ and $y_8^{\rm eff}$ terms. The scale dependence of the predicted ε'/ε is traced back to the scale dependence of the effective Wilson coefficient $y_6^{\rm eff}$ (see Table I). As mentioned before, formally $y_6^{\rm eff}$ should be scale independent to the order α_s. It is thus not clear to us why $y_6^{\rm eff}$(NDR) and $y_6^{\rm eff}$(HV) are not the same to the accuracy under consideration. Furthermore, the scale dependence of $y_6^{\rm eff}$ is amplified by the strong cancellation between QCD penguin and electroweak penguin contributions, which makes it difficult to predict ε'/ε accurately. It appears to us that the different results of ε'/ε in NDR and HV schemes can be regarded as the range of theoretical uncertainties. It is easily seen that a suppression of $B_8^{(2)}$ or an enhancement of $B_6^{(0)}$ will render ε'/ε larger; that is, a ratio of $B_6^{(0)}/B_8^{(2)}$ greater than unity will help get a large ε'/ε. However, in our approach $B_8^{(2)} \sim B_6^{(0)} = 1.5$ and they are nearly scheme independent. We have also studied the dependence of ε'/ε on the nonfactorized effect χ_{LR} and found that it increases slowly with χ_{LR} (see Fig. 5), opposite to the case of A_0/A_2.

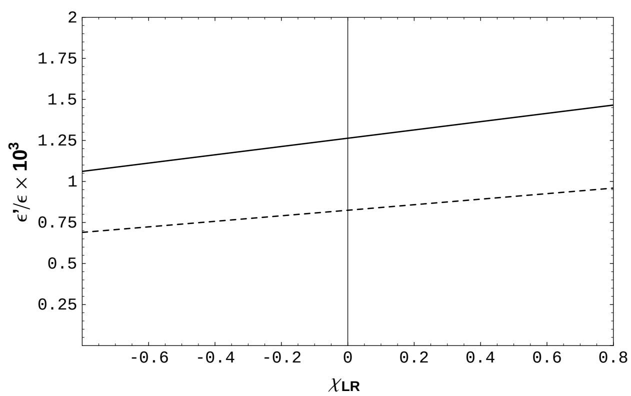

FIG. 5. Direct CP violation ε'/ε versus χ_{LR} for ${\rm Im}(V_{td}V_{ts}^*) = 1.29 \times 10^{-4}$ and $m_s(1\,{\rm GeV}) = 140$ MeV, where the solid (dotted) curve is calculated in the NDR (HV) scheme.

Since the $\Delta I = 1/2$ rule and ε'/ε are both under-estimated theoretically, it is natural to ask if there exists a strong correlation between them. The two principal mechanisms responsible for the enhancement of A_0/A_2 are the vertex-type as well as penguin-type corrections to the matrix elements of four-quark operators, and the nonfactorized effect due to soft-gluon exchange. Turning off these two effects by setting $\chi_{LL} = \chi_{LR} = 0$ and $y_i^{\rm eff} \to y_i^{\rm LO}$, we find that ε'/ε does not get changed in a significant way. On the other hand, if a small strange quark mass is responsible for the remaining enhancement necessary for accommodating the data of A_0/A_2, it turns out that $m_s(1\,{\rm GeV}) = 85$ MeV and $\varepsilon'/\varepsilon = (2.3 - 3.5) \times 10^{-3}$. However, this m_s is too small even compared to the recent lattice result [27] which favors a lower strange quark mass: $m_s(2\,{\rm GeV}) = (84 \pm 7)\,{\rm MeV}$.

VI. CONCLUSIONS

The $\Delta I = 1/2$ rule and direct CP violation ε'/ε in kaon decays are studied within the framework of the effective Hamiltonian approach in conjunction with generalized factoriza-

tion for hadronic matrix elements. Our results are as follows.

1. We identify two principal sources responsible for the enhancement of $\mathrm{Re}A_0/\mathrm{Re}A_2$: the vertex-type as well as penguin-type corrections to the matrix elements of four-quark operators, which render the physical amplitude renormalization scale and scheme independent, and nonfactorized effect due to soft-gluon exchange, which is needed to suppress the $\Delta I = 3/2$ $K \to \pi\pi$ amplitude. This approach is not only much simpler and logical than chiral loop calculations but also applicable to heavy meson decays.

2. We obtain renormalization-scheme independent predictions for $K \to \pi\pi$ amplitudes and find $\mathrm{Re}A_0/\mathrm{Re}A_2 = 13 - 15$ if $m_s(1\,\mathrm{GeV})$ lies in the range $(125-175)\,\mathrm{MeV}$. The tree-level current$\times$current operators account for around 85% of $\mathrm{Re}A_0$. We conjecture that the W-exchange mechanism may provide an additional important enhancement of the $\Delta I = 1/2$ amplitude.

3. The bag parameters B_i, which are often employed to parametrize the scale and scheme dependence of hadronic matrix elements, are calculated in two different renormalization schemes by considering the vertex-like and penguin-like corrections to four-quark operators. It is found that $B_8^{(2)} \sim B_6^{(0)}$, both of order 1.5 at $\mu = 1$ GeV, are nearly γ_5 scheme independent, whereas $B_{3,5,7}^{(0)}$ and $B_7^{(2)}$ show a sizable scheme dependence. Our results $B_{1,2}^{(2)} = 0.34$ and $B_{9,10}^{(2)} = 0.35$ in the NDR scheme are smaller than the value quoted in the literature since we have taken into account isospin breaking contributions to the $\Delta I = 3/2$ amplitude. As for the dependence of B-parameters on m_s, only $B_{1,3,4}^{(0)}$ exhibit a significant m_s dependence, while the rest B-parameters are almost m_s independent.

4. Nonfactorizable contributions to the hadronic matrix elements of $(V-A)(V-A)$ four-quark operators are extracted from the measured $K^+ \to \pi^+\pi^0$ decay to be $\chi_{LL} = -0.73$, while the nonfactorized term for $(V-A)(V+A)$ operators lies in the range $-0.45 < \chi_{LR} < 0$. We found that $\mathrm{Re}A_0/\mathrm{Re}A_2$ (ε'/ε) decreases (increases) slowly with χ_{LR}.

5. For direct CP violation, the prediction of ε'/ε is renormalization scheme dependent owing to the scale dependence with the effective Wilson coefficient y_6^{eff} for reasons not clear to us. We obtain $\varepsilon'/\varepsilon = (0.7 - 1.1) \times 10^{-3}$ if $m_s(1\,\mathrm{GeV}) = 150$ MeV and $\varepsilon'/\varepsilon = (1.0 - 1.6) \times 10^{-3}$ if m_s is as small as indicated by recent lattice results.

ACKNOWLEDGMENTS

This work is supported in part by the National Science Council of the Republic of China under Grant No. NSC89-2112-M001-016.

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